vided by Barker's study of the TO phonon mode in GaP [A. S. Barker, Phys. Rev. 165, 917 (1968)]. The shape of the large-angle Raman line and IR absorption peak deviate strongly from the Lorentzian shape. Barker has carried out an analysis of his data by introducing a frequency-dependent damping factor. By studying the shape of the polariton spectrum of this mode, in principle one may extend Barker's analysis to a much wider range of frequencies. In essence, in the polariton regime, one can sweep the line center over a wide frequency region by varying the scattering angle.

[5]L. Van Hove, Phys. Rev. 95, 249 (1954); 95, 1374 (1954).

[6]J. J. Hopfield, Phys. Rev. 112, 1555 (1958).

[7]For example, see E. Burstein, S. Ushioda, A. Pinczuk, and J.F. Scott, 1969), paper A-1. in *Proceedings of the International Conference on Light Scattering Spectra of Solids, New York*, 1968, edited by G. B. Wright (Springer, New York, 1969).

[8]See the thesis of S. Ushioda cited in Ref. 1. Our expression for the scattering efficiency is one-half that given by Ushioda. We believe Ushioda's result is in error, because he presumed the cross section associated with both the Stokes and anti-Stokes line (in the classical theory) is proportional to the mean-square value $\langle u^2\rangle_T$ of the *total* relative displacement of the ions in the unit cell. In fact, $u(t)$ contains a positive frequency component u_+ and a negative frequency part u_-, with (in the classical theory) $\langle u_+^2\rangle_T = \langle u_-^2\rangle_T = \frac{1}{2}\langle u^2\rangle_T$. The cross section for anti-Stokes scattering is proportional to $\langle u_+^2\rangle_T$, while that for Stokes scattering is proportional to $\langle u_-^2\rangle_T$, rather than $\langle u^2\rangle_T$. Thus, Ushioda's cross section should be reduced by a factor of 2. This will bring it into agreement with our result.

[9]D. T. F. Marple, J. Appl. Phys. 35, 539 (1964).

[10]B. Tell, S. P. S. Porto, and T. C. Damen, Phys. Rev. Letters 16, 450 (1966).

PHYSICAL REVIEW B VOLUME 1, NUMBER 12 15 JUNE 1970

COMMENTS AND ADDENDA

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in Physical Review Letters and are not appropriate for regular Articles. It includes only the following types of communications: (1) comments on papers previously published in The Physical Review or Physical Review Letters; (2) addenda to papers previously published in The Physical Review or Physical Review Letters, in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section may be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.

Cooling Curve in a One-Dimensional Crystal

K. J. Mork
University of Trondheim, Norges Laererhøgskole, Trondheim, Norway
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A region of $2N+1$ particles in a harmonic one-dimensional crystal is given temperature T_0 at time $t=0$. The central particle in this region cools down as follows[1]:

$$T(t) = T_0 \sum_{n=-N}^{N} J_n^2(Nt) , \qquad (1)$$

where t is a suitable dimensionless time variable. By very complicated asymptotics Rubin[1] was able to determine the following limiting behavior:

$$\lim_{N\to\infty} T(t)/T_0 = 1, \qquad \text{for } t<1$$

$$= 2\pi^{-1}\arcsin(1/t), \quad \text{for } t>1 . \qquad (2)$$

The present note demonstrates how this limiting cooling curve may be found in a simple manner. The standard representation[2]

$$J_n^2(z) = 2\pi^{-1}\int_0^{\pi/2} J_0(2z\sin\phi)\cos 2n\phi\, d\phi$$

allows the summation over n in Eq. (1) to be performed. Introducing a new variable $x = 2N\sin\phi$, we obtain

$$T(t)/T_0 = 2\pi^{-1}\int_0^{2N} dx\, x^{-1} J_0(xt) \times \sin[2N\arcsin(x/2N)] + J_N^2(Nt) . \qquad (3)$$

Taking formally the limit $N \to \infty$ in (3), we have

$$\lim_{N \to \infty} T(t)/T_0 = 2\pi^{-1} \int_0^\infty dx\, x^{-1} J_0(tx) \sin x ,$$

equal to[2] the right-hand side of Eq. (2). The formal step involved is easily justified but details are omitted here.

[1]R. J. Rubin, Phys. Rev. 131, 964 (1963). See in particular his Appendix C.

[2]W. Magnus and F. Oberhettinger, *Formulas and Theorems for the Special Functions of Mathematical Physics* (Chelsea Publishing Co., New York, 1949), pp. 32–34.

PHYSICAL REVIEW B VOLUME 1, NUMBER 12 15 JUNE 1970

Specular Reflection in Al Films

W. F. Druyvesteyn

Philips Research Laboratories, N. V. Philips' Gloeilampenfabrieken, Eindhoven, Netherlands

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Experimental results on the size-affected resistivity of aluminum films are discussed.

We wish to comment on two conclusions reached by von Bassewitz and Mitchell[1] (BM) from their experimental results:

(a) The value of the mean free path l, for which they give a value of 17.5 μ for polycrystal films as well as for single-crystal films.

(b) The reflection coefficient p, BM claim that the reflection at a smooth surface of a single-crystal film of Al is nearly specular.

1. DETERMINATION OF l

To obtain the mean free path l from the measurements of the size-affected resistivity ρ, BM plot $1/\rho t$ (t is the thickness of the film) as a function of $\ln t$ and draw a straight line through the experimental points. Extrapolation of the line of the axis where $1/\rho t = 0$ gives an intersection point $t_1 = 17.5$ μ. BM conclude that the intersection point should be the point where $t_1/l = 1$. This leads to $l = 17.5$ μ and $\rho_\infty l = 11.5 \times 10^{-12}\,\Omega$ cm^2, where ρ_∞ is the bulk resistivity. To obtain this, BM use an approximation of the Fuchs-Sondheimer[2] (FS) theory for the case $t/l \ll 1$ which reads

$$\rho_\infty/\rho = (3t/4l)\,(1+p)/(1-p)\,\ln(l/t) \quad . \qquad (1)$$

If $l = 17.5$ μ, then the experiments of BM were done in a regime $0.01 < t/l < 0.2$. However, in this regime the extrapolation procedure described above is misleading. This is most easily seen by plotting $1/\rho_{th} t$ versus $\ln t$, where ρ_{th} is the (numerical) value of the full FS theory. When a straight line is drawn through the calculated points in the regime $0.01 \leq t/l \leq 0.2$, one finds that extrapolation to $1/\rho_{th} t = 0$ gives $t_i/l = 1.9$ if $p = 0$, $t_i/l = 1.3$ if $p = 0.1$, $t_i/l = 0.9$ if $p = 0.3$, and $t_i/l = 0.75$ if $p = 0.5$, instead of $t_i/l = 1$ as follows from Eq. (1). These differences clearly show that if $l = 17.5$ μ, Eq. (1) may not be used. The procedure used by BM to obtain $l = 17.5$ μ and thus $\rho_\infty l = 11.5 \times 10^{-12}\,\Omega$ cm^2 is, in fact, inconsistent with the FS theory.

2. DISCUSSION OF REFLECTION COEFFICIENT p

When concluding something about the reflection coefficient p from the FS theory, the dependence of l on t must be known. As BM have done no other experiments from which l can be determined, they had to make an assumption about the dependence of l on t. In fact, they reason in terms of a constant mean free path and then conclude from the fact that the slope of the straight line in the $1/\rho t$-versus-$\ln t$ graph is different for single-crystal films and polycrystal films that $p \neq 0$. Using Eq. (1) they found that for smooth surfaces of single-crystal films $0.42 < p < 0.84$. From the discussion given above about the validity of Eq. (1) for the experiments of BM, it is clear that the obtained value of p seems not to be correct. The conclusion that $p \neq 0$ can, however, be right if indeed l is independent of t. It is clear, however, that one could also interpret the experiments by assuming $p = 0$ and letting l be a function of t, a possibility which is not at all unlikely in view of the results obtained by Cotti *et al.*[3] Let us consider the consequences of such an interpretation; BM give that ρ varies from 8.5×10^{-8} to 1.1×10^{-8} Ω cm for single-crystal films, and from 18×10^{-8} to 2.0×10^{-8} Ω cm for polycrystal films for thicknesses between 0.2 and 3.6 μ. Using the results of a two-band model,[4] one finds that for single-crystal films, l varies from 6.1 to 18 μ and from